First Midterm

MA441: Algebraic Structures I

8 October 2003

Please include the Honor Code pledge in your test booklet:

> I understand and will uphold the ideals of academic honesty as stated in the Honor Code.

1) Definitions

1. Define what it means for a subset of a group to be a **subgroup**.

2. Define what it means for a group to be **cyclic**.

2) Give an example of

1. a nonabelian group of order 10.

2. a group with exactly five subgroups (including the trivial subgroup and itself). List the subgroups.

3) Fill in the blanks.

1. The order of 3 in $U(11)$ is __.

2. The order of the group $U(15)$, which equals $\phi(15)$, is __.

3. If $x \in G$, $x \neq e$, and $x^{18} = x^{33} = e$, then $|x| =$ __.

4. In the group D_4, let R denote rotation by 90 degrees counterclockwise, and let F denote a flip about the vertical. Written in the form $R^i F^j$, the element FR equals __.

5. A complete list of all generators of $\mathbb{Z}/10\mathbb{Z}$ is __.

4) Euclidean algorithm

1. Use the Euclidean Algorithm to express $\gcd(57, 5)$ as an integer linear combination of 57 and 5. Show your work.

2. Find the multiplicative inverse of 5 in $U(57)$. Show your work.

5) Permutations
Consider permutations of the set $\{1, 2, 3, 4, 5, 6, 7\}$.

1. Let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 4 & 2 & 6 & 5 & 1 \end{pmatrix}, \ \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 7 & 3 & 1 & 4 & 6 \end{pmatrix}.$$

   Write α and β in cycle notation.

2. Compute the composition $\alpha\beta$ and write it in cycle notation. Write α^{-1} in cycle notation. (Note: we compose permutations from left to right, so $(123)(12) = (23)$, not (13).)

The next two questions ask for proofs. Be sure to write carefully and explain your arguments with clear and coherent sentences.

6) Let a be an element of a group G. Define $\langle a \rangle$, the cyclic subgroup generated by a, and prove that it is a subgroup of G.

7) Let G be a group. Show that

$$Z(G) = \bigcap_{a \in G} C(a),$$

that is, the center of a group is the intersection of the centralizers of every element in the group.